NEL/REPORT 1428

SEA-SURFACE TEMPERATURE ESTIMATION

Autocorrelation, regression, and trend analyses of six sea-surface temperature time-series

C. J. Van Vliet and E. R. Anderson • Research Report • 17 January 1967
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PROBLEM

Develop statistical, physical, and computer techniques for interpreting, summarizing, and extrapolating oceanic and meteorologic data for reliable estimation of the sound velocity distribution in the ocean. Specifically, perform autocorrelation, regression, and trend analyses of six time-series of daily sea-surface temperatures. Compensate for missing data in the time-series. Examine the randomness of mean annual temperatures, and of amplitudes and phases descriptive of annual variations in temperature.

RESULTS

Analysis of records of sea-surface temperatures taken in the North Atlantic and North Pacific and up to 40 years in length has led to the following conclusions:

1. The autocorrelation statistics indicate the existence of an oscillatory function with period 1 year in the records and, for most stations, an oscillatory function with period 0.5 year. There is no evidence of functions with shorter periods.

2. For all stations considered, a regression model containing annual and semiannual oscillatory terms (sines and cosines) provides a good statistical fit to the observed daily temperatures. The analysis compensates for missing data.

3. No trends exist in the sequences of annual mean temperatures, or in the sequences of amplitudes and phases describing the regression functions. However, there are significant differences among the annual mean temperatures. The behavior of the annual means, amplitudes, and phases is typical of random statistical variables.

RECOMMENDATIONS

Examine the results of this report from an oceanographic point of view. Specifically, answer the following questions:

1. What is the length of time-series necessary to produce reliable long-term estimates of sea-surface temperatures?

2. Can it be expected that sea-surface temperatures will vary systematically over periods of several years, in light of the randomness demonstrated in this report?

ADMINISTRATIVE INFORMATION

Work was performed under SR 104 03 01, Task 0586 (NEL L40561). The report covers work from July 1961 to August 1964 and was approved for publication 17 January 1967.

The assistance of Dr. George W. Snedecor, in providing motivation, encouragement, and advice; of Mrs. J. M. Baker, J. S. Buehler, and H. W. Frye, for numerical analysis assistance; and of Mrs. G. L. Jones, for handling the many details essential to the success of such a study, is gratefully acknowledged.

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INTRODUCTION

This study is the third in a series of studies concerned with the analysis of sea-surface temperature observations. The first study dealt with the effect of missing data in long time-series of sea-surface temperature measurements on certain regression and autocorrelation analyses.[1] The second study examined the use of regression models for time-space interpolation of sea-surface temperature observations.

This study presents the results of an autocorrelation, regression, and trend analysis of time-series of sea-surface temperature measurements made at six locations representing different oceanographic conditions, and considers the difficulties encountered in applying these techniques to oceanographic data samples.[2] An oceanographic interpretation of the statistical results will be presented in a later study.

Many time-series measurements have been made at various locations. In the eastern Pacific Ocean such measurements have been made by Canadian and American oceanographers at coastal, island, and ship locations for time periods up to 45 years. These data have been the subject of numerous papers including, among others, those of Pickard and McLeod[3] and Roden.[4,5] This study differs from those cited in that the original daily temperatures are used in the analysis without a preliminary smoothing by monthly averaging.

The purpose of time-series analysis is to isolate *trend*, *oscillation*, and *random elements*, which are defined as follows. Trend is a gradual increase or decrease in a system over a long period of time; an oscillation is a variation about the trend that occurs with more or less regularity over some time interval; and a random element is an unpredictable variation in the variable. If long-term trend does not exist, then the primary need is the statistical fitting of some function to time-series to represent the oscillatory element.

Several sets of daily sea-surface temperatures have been examined, covering two open ocean and four island or coastal locations (fig. 1). To indicate how individual temperature measurements vary throughout the year, 1 year of measurements for each location is presented in figure 2. These years of temperatures are taken from records that vary in length from 7 to 40 years. Pertinent information about the stations yielding these records is summarized in table 1.

The data for station PAPA are being collected by Canadian oceanographers of the Pacific Oceanographic Group, Nanaimo, British Columbia, and are available as a sequence of data reports. Bathythermograph observations are made at 0200 and 1700 GMT. The 0200 GMT data were used for this analysis if the ship were within a 10-minute rectangular area centered at the nominal position.

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[1]Superscript numbers denote references in the list at the end of this report.

TABLE 1. LOCATION OF SEA-SURFACE TEMPERATURE TIME-SERIES

Location	Time Period	Number Days	Number Daily Observations	Percent Possible Daily Observations
Weather Ship PAPA 50°N 145°W North Pacific	1/56 - 8/62 6 yr 7 mo	2409	1595	66
Weather Ship ECHO 35°N 48°W North Atlantic	9/49 - 9/56 7 yr	2557	1533	60
Cape St. James 52°N 131°W North Pacific	1/35 - 1/61 21 yr (5 yr missing)	7671	6180	81
Triple Island 54°N 131°W North Pacific	1/40 - 1/61 21 yr	7671	7264	95
Langara Island 54°N 133°W North Pacific	1/41 - 1/61 20 yr	7304	6402	88
Scripps Pier 33°N 117°W North Pacific	1/21 - 1/61 40 yr	14610	14352	98

If the 0200 GMT data were not available, other data for a given day were used. The station ECHO data are also taken by bathythermograph. The temperature measurements are probably not as accurate as the PAPA data. The Cape St. James, Triple Island, and Langara Island data were taken by lighthouse keepers, and were made within the hour prior to daytime high tide. The Scripps Pier data were collected by the Scripps Institution of Oceanography.

A subjective examination of the data shows, for the PAPA, ECHO, Cape St. James, and Triple Island observations, maximums in August or September and rather flat minimums in February to May. The Langara Island and Scripps Pier data show a more regular sinusoidal seasonal variation. At all stations there appear oscillations with a duration of a few days to a few weeks at irregular intervals. At the open-ocean locations these shorter-period oscillations occur less frequently and their magnitude is smaller than at coastal locations. It is recognized that there exists in the data another oscillation – a diurnal variability. The amplitude of this diurnal oscillation is much smaller than that of the seasonal oscillation and will not be examined in this study. However, it must be recognized as a factor in the overall variability.

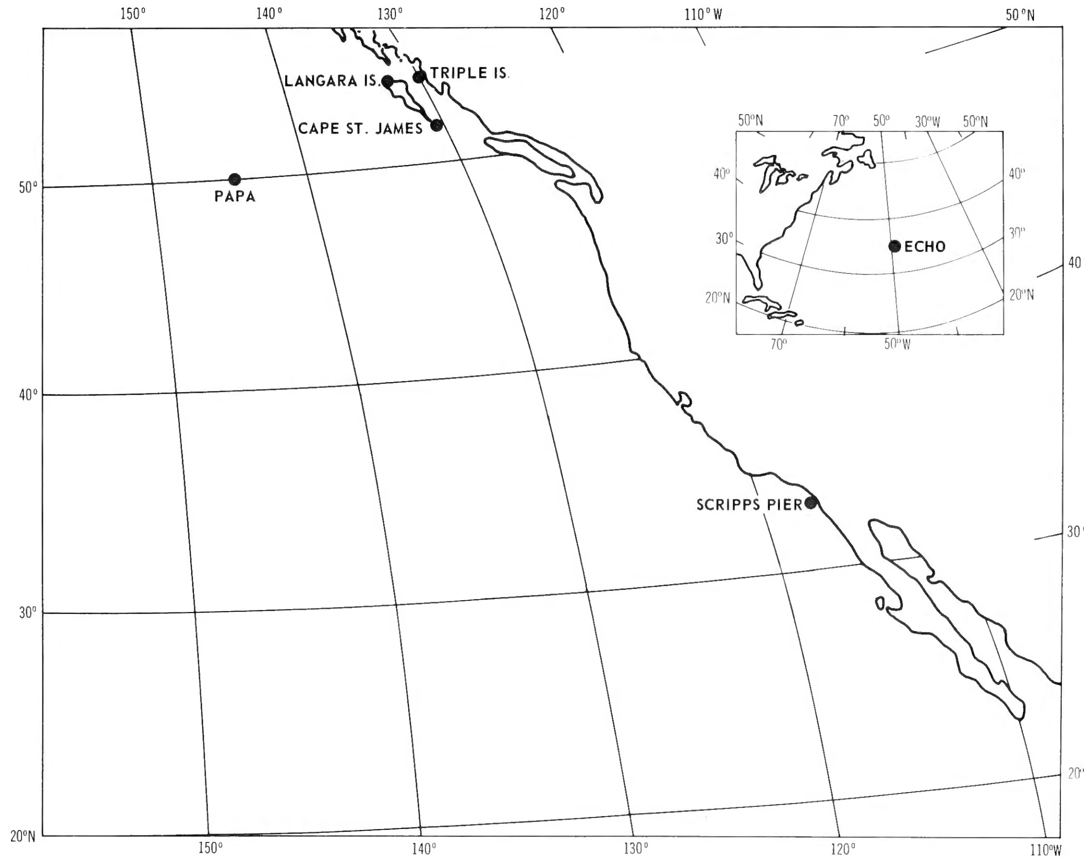

Figure 1. Geographical location of oceanographic stations.

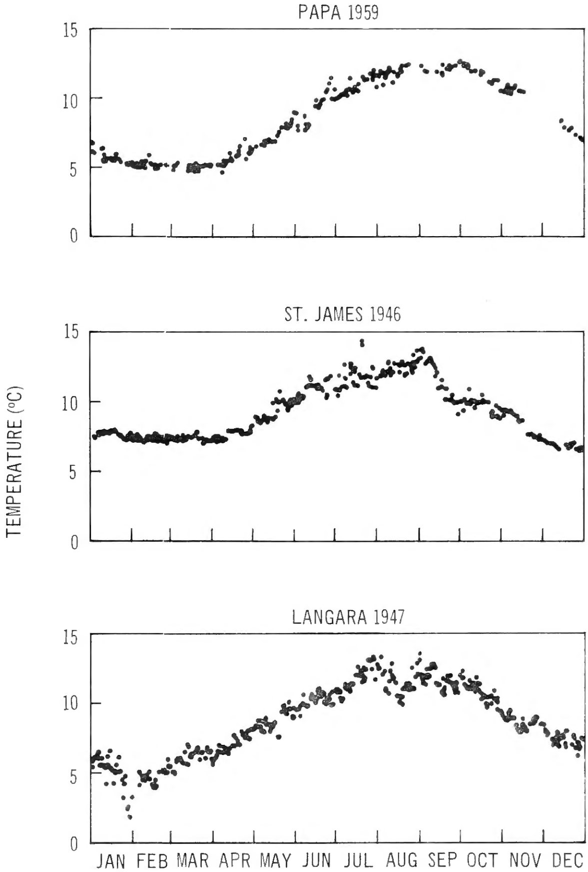

Figure 2. Sea-surface temperatures as a function of time for selected years of data.

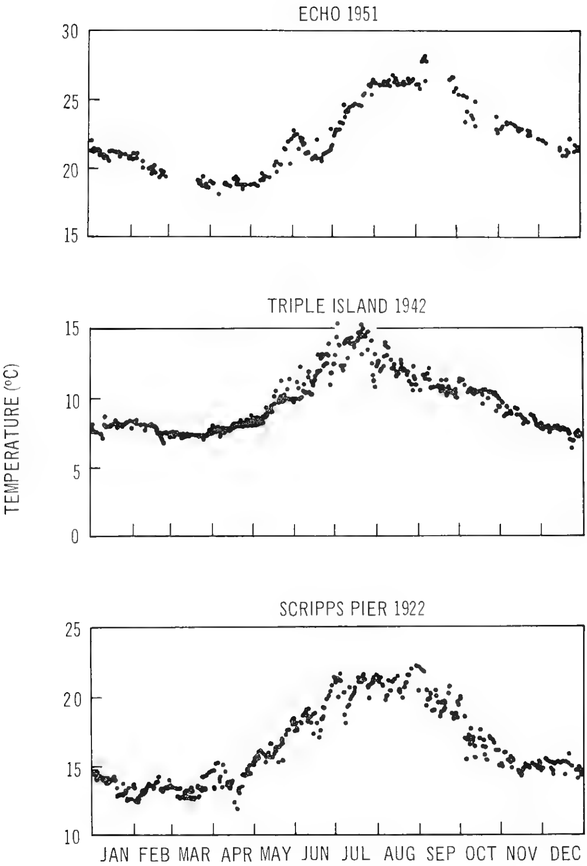

Figure 2. (Continued)

AUTOCORRELATION ANALYSIS

A visual observation of the data suggests statistically fitting some theoretical function which oscillates with period 1 year. Further justification is provided by the autocorrelation function:

$$C_k = \mathrm{COV}\left(T_i T_{i+k}\right) / \left[\mathrm{VAR}\left(T_i\right) \mathrm{VAR}\left(T_{i+k}\right)\right]^{1/2}, \text{ for lags } k = 0, 1, 2, \ldots$$

The variable T_i is the sea-surface temperature on day i, T_{i+k} is the temperature k days later, and COV and VAR are the covariance and variance of the variables as indicated.

Figures 3, 4, and 5 present the results of an autocorrelation analysis for the six time-series. The upper figure for each station is the autocorrelation function of the daily temperatures. The peaks in the autocorrelation functions have magnitudes and spacings indicating so strongly the existence of an annual oscillation in the time-series that any statistical test of hypothesis is superfluous.

The middle set of figures presents the autocorrelations of the residuals (or anomalies) after removing the 12-month oscillatory terms from the original data (discussed in the next section). An obvious feature of these figures is the peaks and troughs in the autocorrelation function at intervals of 6 months for PAPA, ECHO, Cape St. James, and Triple Island, and the lack of this oscillation for Langara Island and Scripps Pier.

The lower set of figures presents the autocorrelation of the residuals after the annual and semiannual oscillatory terms are removed. The autocorrelation function has a form typical for such residuals, decreasing as a negative exponential function for small lags.

The autocorrelation function for Scripps Pier was computed out to a lag of 1800 days, an arbitrary figure slightly over 10 percent of the total sample length. Since conclusions based only on sampling variability of the autocorrelation function must be avoided, values of the function significantly different from zero at some probability level are of primary interest. If the standard deviation of the autocorrelation coefficients were known, and normality assumptions made, then significance levels could be determined. The Scripps Pier autocorrelation coefficients with lags from 400 to 1800 days provide an estimate of σ_c, the standard deviation of the nonsignificant correlations. The estimate is based on a large sample of the autocorrelation coefficients, and the maximum lag involved is still only a small fraction of the total time-series length. The PAPA and ECHO series are much too short to supply such an estimate, and the records for the other stations are of marginal length for this purpose.

This standard deviation is $\sigma_c = 0.0293$. The 95 percent significance levels for Scripps Pier are $\pm 1.96 \times 0.0293 = \pm 0.0574$, for a nominal series length of 13140.

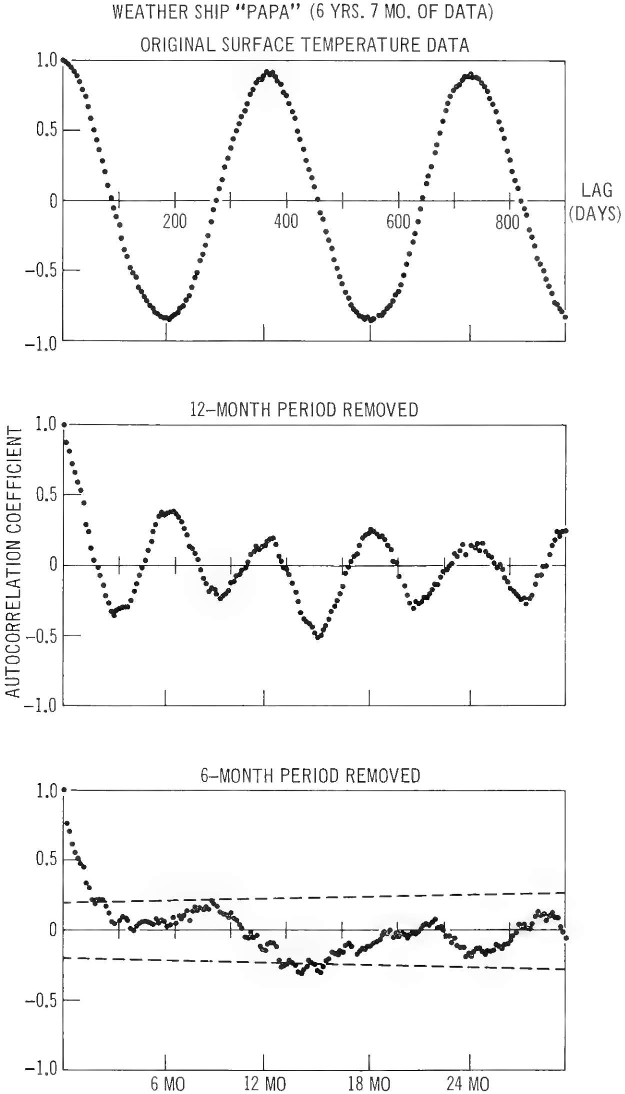

Figure 3. Autocorrelation coefficients for PAPA and ECHO.

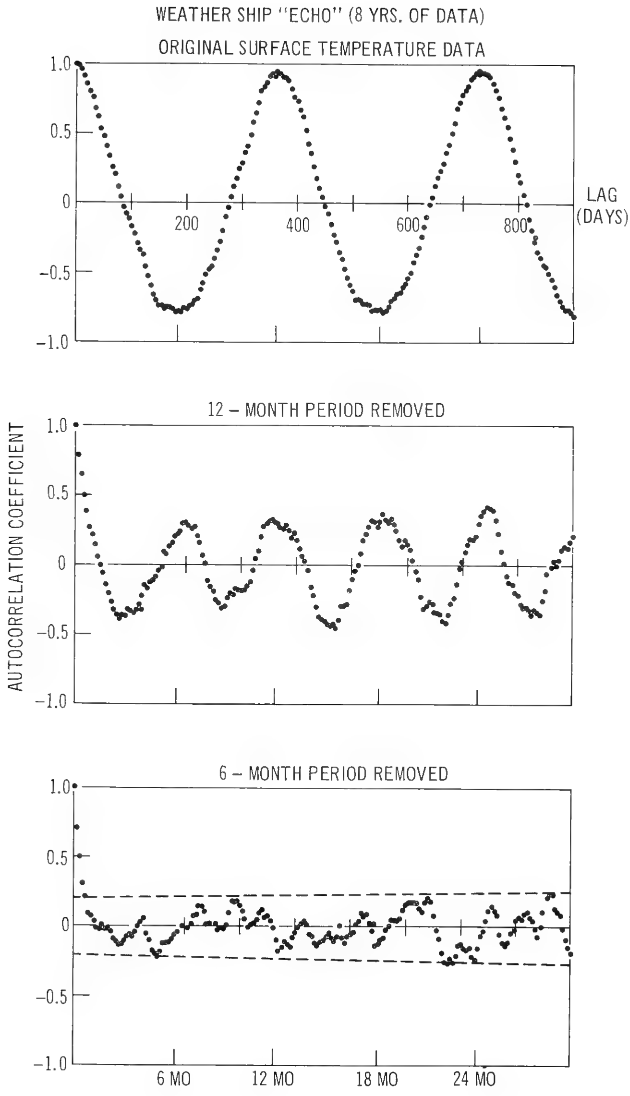

Figure 3. (Continued)

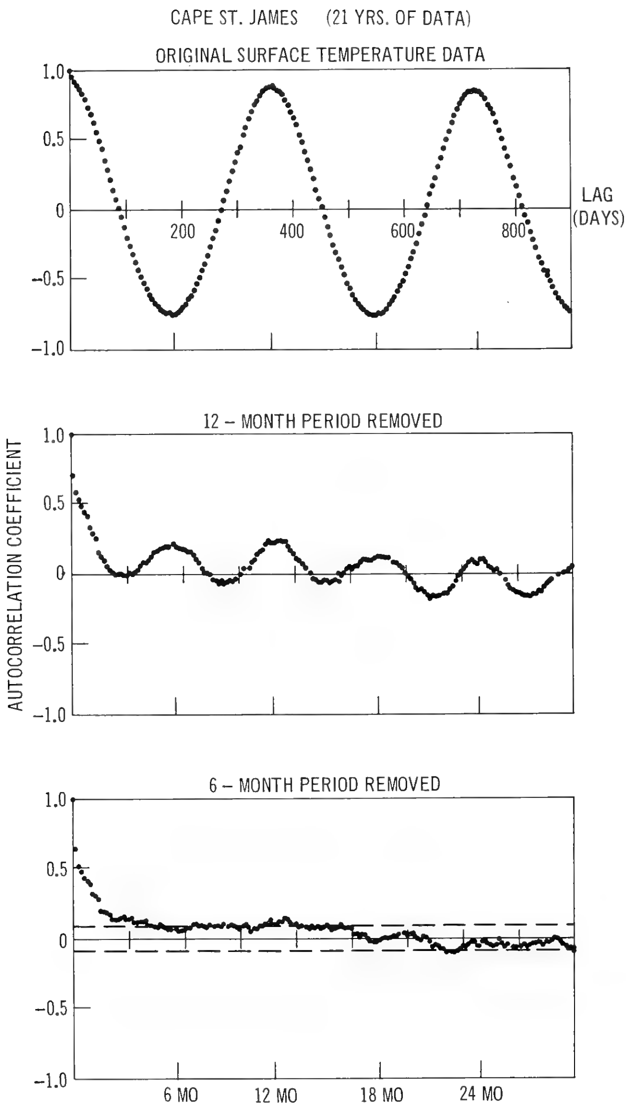

Figure 4. Autocorrelation coefficients for Cape St. James and Triple Island.

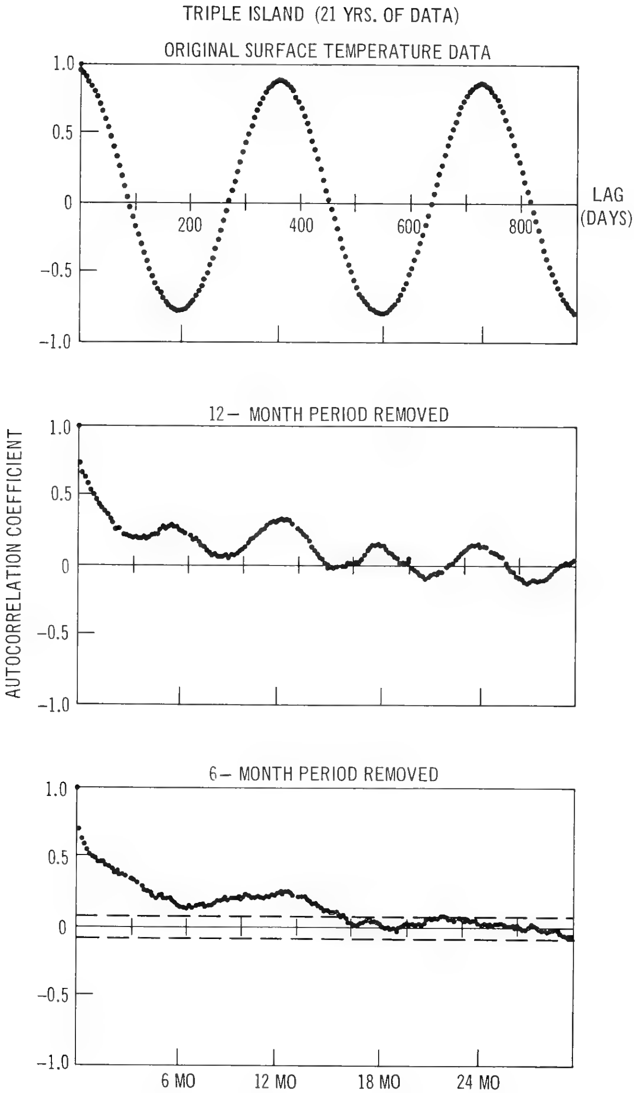

Figure 4. (Continued)

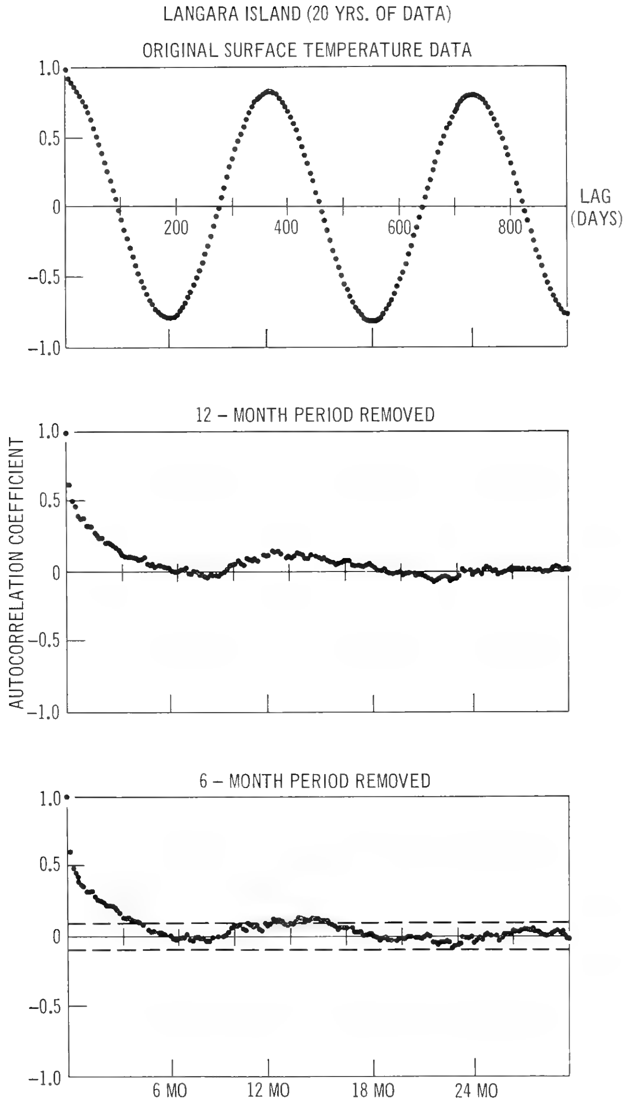

Figure 5. Autocorrelation coefficients for Langara Island and Scripps Pier.

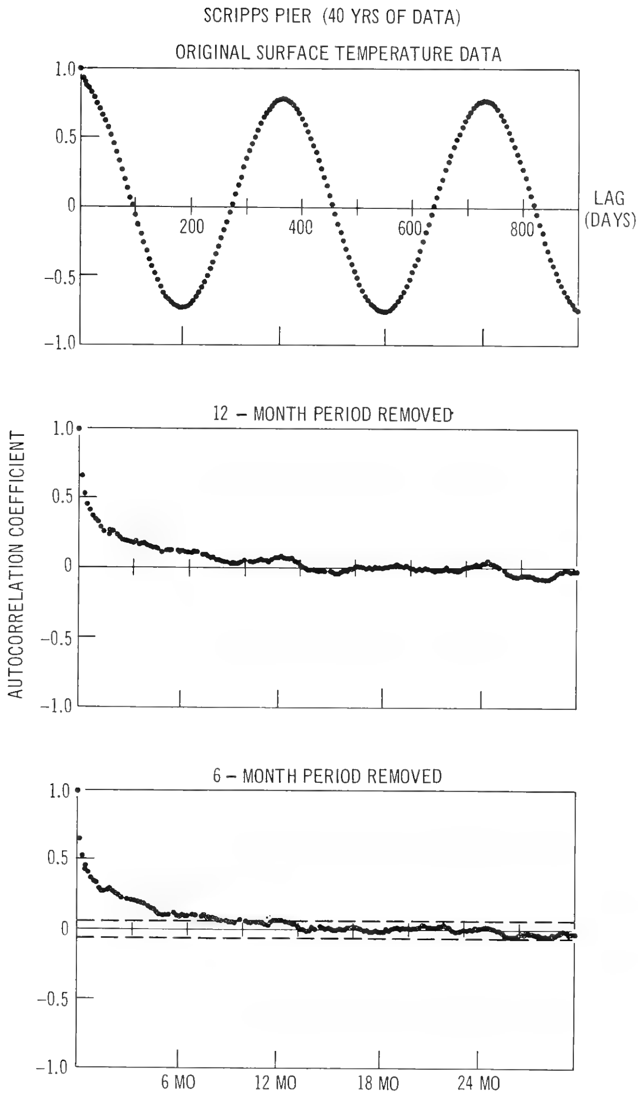

Figure 5. (Continued)

With some caution, this estimate is used for each location. The 95 percent significance levels are displayed as the dashed lines in figures 3, 4, and 5. The limits are adjusted for the particular series length involved and diverge with increasing lag, since the corresponding sample size decreases. The many oscillations in the autocorrelation functions beyond 100 to 200 days' lag are not significant for the lower curves of figures 3, 4 and 5. The oscillations are merely characteristics of the particular samples of time-series available, and it is useless to subject them to any additional correlation or spectral analysis.

Of interest is the comparison of the above standard deviation with that resulting from a sometimes used inequality concerning the true variance of the autocorrelation coefficient.[6] This inequality is

$$\sigma_c^2 \leqslant \frac{4}{T} \int_0^\infty \rho^2(\tau)\, d\tau$$

where $\rho(\tau)$ is the true autocorrelation function, and T is the sample length. If the Scripps Pier empirical autocorrelation function is numerically integrated out to a lag of 350 days, and if the function is assumed to be zero beyond that lag, an estimate of the inequality $\sigma_c^2 \leqslant 0.004424$ is obtained. Correspondingly, $\sigma_c \leqslant 0.0665$. The value $\sigma_c = 0.0293$ easily satisfies the inequality.

REGRESSION ANALYSIS

The preceding autocorrelation analysis indicates that the surface-temperature time-series contains a prominent oscillatory term with a period of 12 months, and that four of the time-series contain an additional oscillatory term with a period of 6 months.

A general model containing oscillatory functions is

$$T' = \beta_0 + \sum_{i=1}^{k} \alpha_i \sin\left[2\pi i\left(D - \theta_i\right)/365\right] + \epsilon, \text{ or expanding,} \tag{1}$$

$$= \beta_0 + \sum_{i=1}^{k} \left[\beta_{2i-1} \sin(2\pi i D/365) + \beta_{2i} \cos(2\pi i D/365)\right] + \epsilon \tag{2}$$

where D is time measured in days from some arbitrary origin, and T' is the fitted value of the surface temperature. Fitting the function of equation (2) to the observed surface temperatures T using the method of least squares yields estimates of the regression coefficients β and an estimate of the variance of ϵ. The

amplitudes α and phases θ can be obtained from the β's. The quantity ϵ is the random error or residual term.

For $k = 1$, equation (2) was fitted to each of the six sets of surface temperature data. The results are shown in table 2.

TABLE 2. HARMONIC ANALYSIS OF ANNUAL OSCILLATION

Location	Regression Coefficients			Amplitude	Phase
	β_0	β_1	β_2	α	θ
PAPA	8.54	-3.51	-1.81	3.95	154.9
ECHO	22.06	-3.66	-1.70	4.03	157.4
Cape St. James	9.11	-2.03	-1.84	2.74	139.9
Triple Island	9.23	-2.23	-2.26	3.18	136.6
Langara Island	8.64	-2.13	-1.97	2.90	139.2
Scripps Pier	16.91	-1.93	-2.60	3.24	128.4

Certain measures related to the statistical fit are given in table 3. The quantity R is the multiple correlation coefficient; σ_0 and σ_D are the standard deviations of the observations about their mean and about the fitted regression curve, respectively. The F-ratio indicates whether or not the regression curve has significantly reduced the total sum of squares.

TABLE 3. STATISTICAL FIT OF ANNUAL OSCILLATION

Location	R^2	σ_0	σ_D	σ_D*	F-ratio	F-ratio*	$F_{0.01}$
PAPA	0.88	2.99	1.05	1.21	811	605	4.6
ECHO	0.89	2.98	0.99	1.17	876	626	4.6
Cape St. James	0.79	2.19	1.00	1.09	553	465	4.6
Triple Island	0.82	2.49	1.06		774		4.6
Langara Island	0.79	2.31	1.05	1.13	612	546	4.6
Scripps Pier	0.76	2.63	1.28		569		4.6

*Adjusted for nonrandom missing data

Van Vliet[1] has shown that the variances of regression coefficients should be increased if there are nonrandom missing data. A similar situation

exists with the residual variances. For a fraction f of missing data, the fractional increase in regression coefficient variance attributable to nonrandom missing data is

$$Q = 2f/(1-f)$$

The corresponding fractional increase in the residual variance is

$$(Q+1)(1-f) - 1 = f$$

Locations PAPA, ECHO, Cape St. James, and Langara Island have nonrandom missing data. A correction for such data is reflected in the columns with the asterisks of table 3. The corrections yield more conservative estimates of σ_D and F.

The analysis of variance leading to the F-ratio is as follows. Assume that there are N complete years of data. Since the sine and cosine functions yield an integral number of periods per year, the N years of data can be interpreted as 1 year of data with N observations per day. One aspect of this fact is that the 5 years of data missing for Cape St. James, as noted in table 1, are not pertinent to the regression analysis. For simplicity, all years are assumed to have 365 days. The total sum of squares about the sample mean can be partitioned as

$$\sum_{i=1}^{N}\sum_{j=1}^{365}\left(T_{ij}-\overline{T}\right)^2 = N\sum_{j=1}^{365}\left(T_j'-\overline{T}\right)^2 + \sum_{i=1}^{N}\sum_{j=1}^{365}\left(T_{ij}-T_j'\right)^2$$

Where T_{ij} is the observed temperature on the j-th day of the i-th year, and T_j' is the temperature predicted by equation (2) for the j-th day. For $k = 1$, the quantity

$$F = \frac{\sum_j \left(T_j'-\overline{T}\right)^2/2}{\sum_i \sum_j \left(T_{ij}-T_j'\right)^2/(365N-3)}$$

has the F-distribution with 2 and $365N$-3 degrees of freedom. Assuming the F-test is robust with respect to missing data, the test is applied in the present situation with $365N$ replaced by the actual number of observations.

The least squares method is valid if (1) the error between the true regression curve and the observed value is distributed independently of the independent variables with zero mean and constant variance; and (2) ideally, successive errors are distributed independently of one another. Actually, the problem of using the method of least squares when the error terms are autocorrelated has been solved if the ϵ's follow certain autoregressive processes.[7]

The residuals, after the annual terms are removed, constitute a time-series with another strongly oscillatory term. If the F-ratios in table 3 were

examined, this fact would at least necessitate reducing the degrees of freedom in the denominator, in turn reducing the actual F-ratio value. In this case a detailed examination of these F-ratios is superfluous, since the reduction would have to be considerable before significance was marginal.

To examine the semiannual oscillations, equation (2), for $k = 2$, was fitted to each of the six sets of surface temperature data. The results are summarized in tables 4, 5, and 6.

TABLE 4. REGRESSION COEFFICIENTS OF ANNUAL AND SEMIANNUAL OSCILLATIONS

Location	Regression Coefficients				
	β_0	β_1	β_2	β_3	β_4
PAPA	8.51	-3.39	-1.97	0.78	-0.48
ECHO	22.06	-3.56	-1.69	0.77	0.21
Cape St. James	9.11	-2.01	-1.82	0.51	0.14
Triple Island	9.23	-2.23	-2.26	0.32	0.39
Langara Island	8.63	-2.13	-1.97	0.16	-0.16
Scripps Pier	16.91	-1.93	-2.60	0.28	0.08

TABLE 5. AMPLITUDES AND PHASES OF ANNUAL AND SEMIANNUAL OSCILLATIONS

Location	Amplitude, °C		Phase, days	
	(12 mo.) α_1	(6 mo.) α_2	(12 mo.) θ_1	(6 mo.) θ_2
PAPA	3.92	0.92	152.0	16.1
ECHO	3.94	0.80	156.8	- 7.7
Cape St. James	2.71	0.53	139.7	- 8.0
Triple Island	3.17	0.51	136.6	-25.9
Langara Island	2.90	0.22	139.3	23.0
Scripps Pier	3.24	0.29	128.4	- 7.8

TABLE 6. STATISTICAL FIT OF ANNUAL AND SEMIANNUAL OSCILLATIONS. F-RATIOS ATTRIBUTABLE TO SEMIANNUAL OSCILLATION

Location	R^2	σ_0	σ_D	σ_D*	F-ratio	F-ratio*	$F_{0.01}$
PAPA	0.92	2.99	0.84	0.97	65	49	4.6
ECHO	0.93	2.98	0.81	0.96	55	39	4.6
Cape St. James	0.82	2.19	0.93	1.02	24	20	4.6
Triple Island	0.84	2.49	1.00		22		4.6
Langara Island	0.80	2.31	1.04	1.10	3.8	3.4	4.6
Scripps Pier	0.77	2.63	1.27		4.8		4.6

*Adjusted for nonrandom missing data.

The addition of semiannual oscillatory terms to the regression equation improves the fit obtained with the annual terms. Except for Langara Island, the F-ratios are still significant at the 1 percent level, but are much smaller than for annual terms. Because of the marginal significance for Scripps Pier, the question of whether the assumptions about the residuals are satisfied is more pertinent.

TREND ANALYSIS

If the surface temperature for each year could be represented by a single quantity, it might be possible to identify a trend (gradual change in the system) over a period of several years. The regression analysis of the time-series provides in β_0 an estimate of the yearly average of the surface temperature. Figure 6 is a plot of the β_0's obtained by fitting equation (2) to each calendar year of data taken at four locations.

The statistical method chosen to test for trends is that of the theory of runs.[8] The test is one for randomness in a sequence of observations. The only underlying assumption is that the variable under consideration be continuous. The test is performed as follows. The median value for each sequence of β_0's is determined. Each β_0 is assigned the letter A if it is above the median or the letter B if it is below the median, the median β_0 being omitted. A run is defined as a succession of one or more identical letters. The following sequences of runs are obtained.

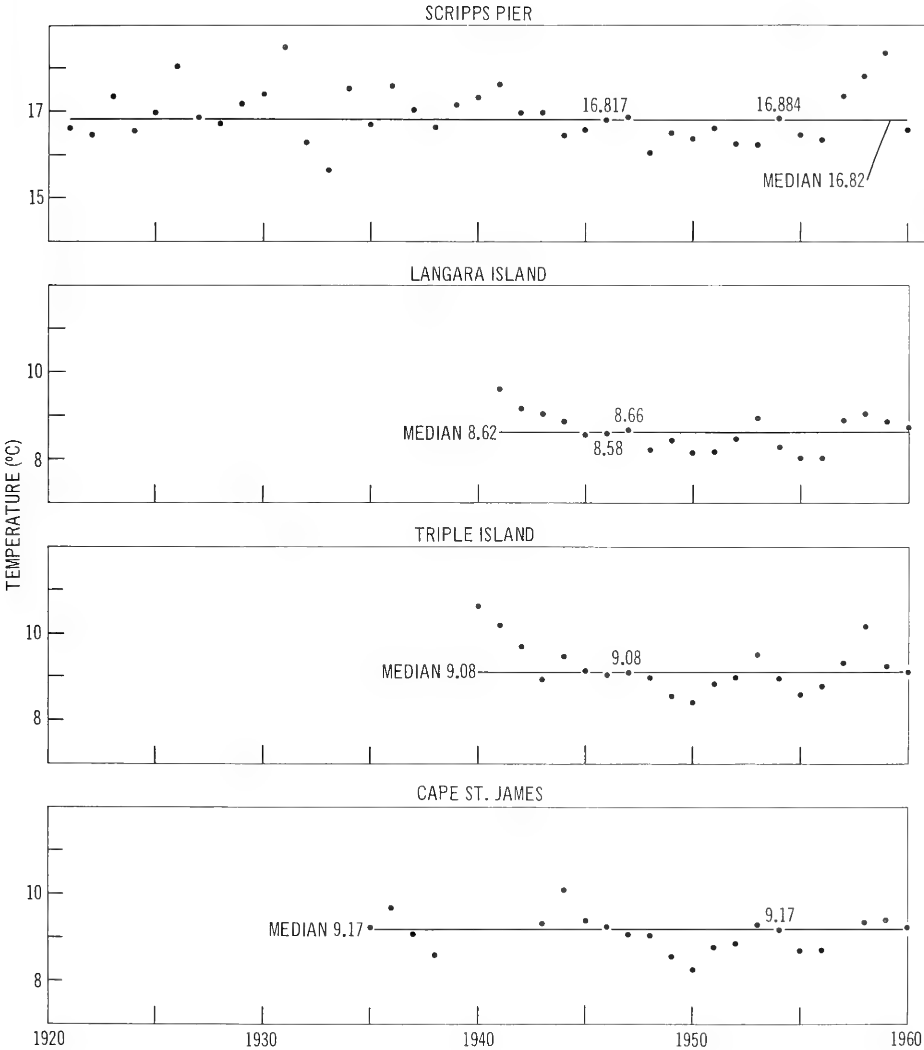

Figure 6. Annual averages, β_0, of sea-surface temperatures for coastal and island stations.

Scripps Pier:

BB A B AAA B AAA BB A B AA B AAAAA BBB A BBBBBB A BB AAA B (19 runs)

Langara Island:

AAAA BB A BBBBB A BBB AAAA (7 runs)

Triple Island:

AAA B AA BBBBBB A BBB AAAA (7 runs)

Cape St. James:

AA BB AAAA BBBBBB A BB AAA (7 runs)

If the β_0's are randomly distributed with respect to time, a fairly large number of runs is expected. If a trend exists in a sequence of β_0's, only a few runs are expected. The theoretical distribution and the critical values of the number of runs can be determined.[8] The critical number of runs at the 5 percent probability level for the 40 observations at Scripps Pier is 15, the null hypothesis of no trend being rejected if there are 15 or fewer. Since the observed number of runs is 19, the null hypothesis is not rejected, and it is concluded that no long-term trend exists in the β_0's for the 40 years of data for Scripps Pier.

Langara Island, Triple Island, and Cape St. James each have 7 runs in 20 observations, the median β_0 being omitted for Triple Island. The probability of 7 or fewer runs arising by chance in 20 observations is 0.051, so the 7 runs are not quite significant at the 5 percent level. It is concluded that no trend exists in the records for any of the three locations.

It should be pointed out that once a time series of 20 to 40 years in length is selected for analysis, runs as long as 5 or 6 years, among those observed, are reasonable and expected. For example, a slightly different test of hypothesis using runs is based on the length of the longest run.[9] For the longest run to be significant at the 5 percent level, it must be at least length 7 in 20 observations or length 9 in 40 observations. The longest runs obtained in this analysis are of length 6, and are not significant for either series length.

An alternate test for randomness is the autocorrelation coefficient with lag 1, or more simply the statistic

$$W = \sum_{i=1}^{N} X_i \, X_{i+1}$$

If a set of observations is ordered with respect to time, and if time is irrelevant, no correlation would be expected to exist between successive pairs of values of

the sequence. A nonparametric test has been devised to test the hypothesis of zero autocorrelation.[10] The random variable W is approximately normally distributed with mean

$$M_W = \frac{S_1^2 - S_2}{N - 1}$$

and variance

$$\sigma_W^2 = \frac{S_2^2 - S_4}{N - 1} + \frac{S_1^4 - 4S_1^2 S_2 + 4S_1 S_3 + S_1^2 - 2S_4}{(N-1)(N-2)} - M_W^2$$

where

$$S_K = \sum_{i=1}^{N} X_i^K$$

The X's are the sequence of β_0's. For Scripps Pier, $N = 40$, and $X_{41} = X_1$. Calculations yield $W = 149.184$, $M_W = 145.850$, and $\sigma_W = 2.25$, so that $t = (W - M_W)/\sigma_W = 1.48$. For t a standard normal variable, the 5 percent critical value is 1.64, since the alternative hypothesis is $W > M_W$. Since $1.48 < 1.64$, the null hypothesis $W = M_W$ is not rejected, and again it is concluded that no trend exists in the sequence of β_0's.

Since no trend was detected in the Scripps Pier data, it is of interest to go one step further into the question of randomness and examine the empirical distribution of certain statistics obtained from the analysis of the 40 individual years. Figure 7A is a histogram of the same set of β_0's that were tested for trend. The normal curve with the sample mean of 16.912 and sample standard deviation of 0.613 is also shown in the figure. Even though the histogram is skewed, a chi-square, goodness-of-fit test leads to an acceptance of normality at the 5 percent probability level.

The purpose of this study is not one of making goodness-of-fit tests, and no further use is made of this technique. Rather it is included to point out that, on the basis of tests for trend and the histogram above, quantities such as β_0 used to characterize sea-surface temperatures for an entire year behave exactly as one expects independent random variables to behave. This is not to deny the existence of real year-to-year differences in the ocean, but rather to emphasize that these differences are not unexpected to an oceanometrician.

As a test for year-to-year differences in the β_0's, table 7 displays an

analysis of variance for each station. The between-years sum of squares is $\sum_{1}^{N} (\beta_0 - \bar{\beta}_0)^2$; the within-years sum of squares is $\sum_{1}^{N} \sigma_D{}^2$. The quantities β_0 and $\sigma_D{}^2$ are individual year values and the summations are over the N years in a sample. Since the off-diagonal terms in the product-moment matrix are negligible, an individual year $\sigma_D{}^2$ divided by the appropriate sample size is an estimate of the variance of β_0 for that year. (A discussion of the variance of regression coefficients can be found in reference 11.) The F-ratios are all highly significant, and it is concluded that there are real year-to-year differences in the β_0's.

TABLE 7. ANALYSIS OF VARIANCE OF ANNUAL β_0

Location	Between Years			Within Years			F-Ratio	
	Sum of Squares	Degrees of Freedom	Mean Square	Sum of Squares	Degrees of Freedom	Mean Square	F	$F_{0.01}$
PAPA	0.912	6	0.1520	1.399	1560	0.00090	169	2.8
ECHO	1.338	6	0.2230	2.691	1498	0.00180	124	2.8
Cape St. James	3.490	20	0.1745	9.434	6075	0.00155	113	1.9
Triple Island	6.393	20	0.3197	9.016	7159	0.00126	254	1.9
Langara Island	3.435	19	0.1808	12.111	6302	0.00192	94	1.9
Scripps Pier	14.659	39	0.3759	35.255	14152	0.00249	151	1.6

Similar analyses of randomness have been applied to the Scripps Pier 40-year time-series of annual amplitude, annual phase, and percent variance explained. Histograms are shown in figures 7B, C, and D. In all cases the histograms are typical of those obtained from a sample of size 40 from a population where the variable in question has a unimodal, slightly skewed frequency function. In addition the run tests for trend yield total runs of 18, 23, and 22 for amplitude, phase, and percent variance explained, respectively. All these totals are greater than the still applicable critical value of 15 used for the β_0's, so again it is concluded that no trend exists in the time-series of these variables.

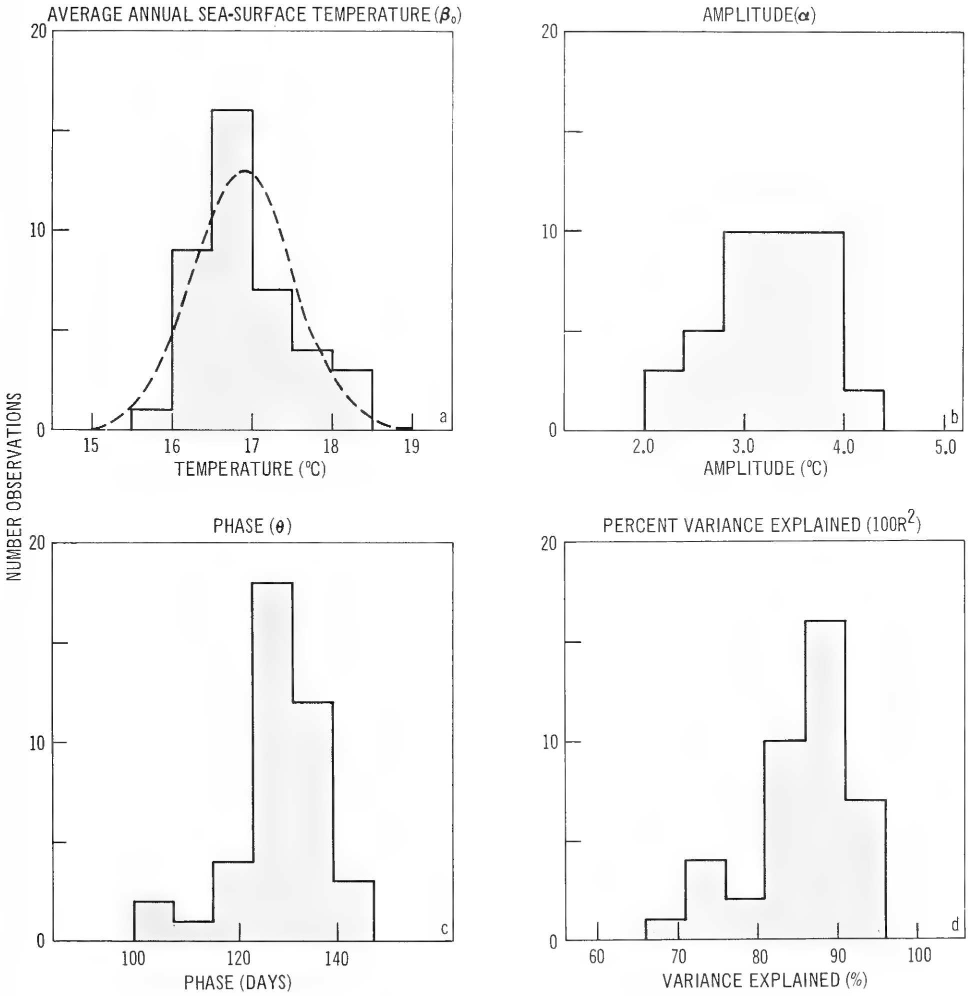

Figure 7. Histograms for annual Scripps Pier data of average temperature, amplitude, phase, and percent variance explained.

SUMMARY AND CONCLUSIONS

By alternating an autocorrelation analysis and an harmonic analysis of certain time-series of daily sea-surface temperatures, an adequate estimator of these temperatures has been determined. A judicious selection of regression variables would have made the autocorrelation analysis unnecessary, but such an analysis does provide clues to the nature of the time-series.

For all stations considered, a regression model containing annual and semiannual oscillatory terms (sines and cosines) provides a good statistical fit to the observed daily temperatures. Some of the stations have nonrandom missing data, generally in the winter. Reference 1 examines in detail the effect of this type data on the variances of regression coefficients and autocorrelation coefficients. These results are extended to the residual variances used in this report. The correction for nonrandom missing data increases the residual variance. Correspondingly, F-ratios are reduced. The effect is conservative. That is, one is less likely to reject null hypotheses after the correction is made than before.

The question of the existence of trends in ocean temperatures is an important one. Several statistical tests for trend were performed on the sequences of annual mean sea-surface temperatures, and on the sequences of amplitudes and phases describing the regression functions. No trends were discovered to exist in any of the sequences. It should be pointed out that, if trends did exist, it would be a straightforward statistical problem to isolate their effect on the time-series.

RECOMMENDATIONS

1. An analysis of variance of the annual mean sea-surface temperatures indicates that there are real year-to-year differences in the means. In the light of these differences it is recommended that an investigation be made into the length of time-series necessary to produce reliable long-term estimates of sea-surface temperatures.

2. The yearly means behave like random variables. Shorter sequences of means with a systematic or cyclical appearance may occur by chance. It is recommended that an investigation be made into the frequency of occurrence of such unusual short sequences.

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13. ABSTRACT

An autocorrelation analysis of six temperature records from the North Pacific and North Atlantic up to 40 years in length showed the existence of an oscillatory function with period 1 year for all the stations studied, and of another oscillatory function with period 0.5 year for most of the stations. A regression model containing annual and semiannual oscillatory terms was found to provide a good statistical fit to the observed daily temperatures. No long-term trends were detected in the sequences of annual mean temperatures, but there were significant differences among these temperatures.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Ocean Surface - Temperature - North Atlantic Ocean						
Ocean Surface - Temperature - North Pacific Ocean						
Ocean Surface - Temperature - Statistical Analysis						

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